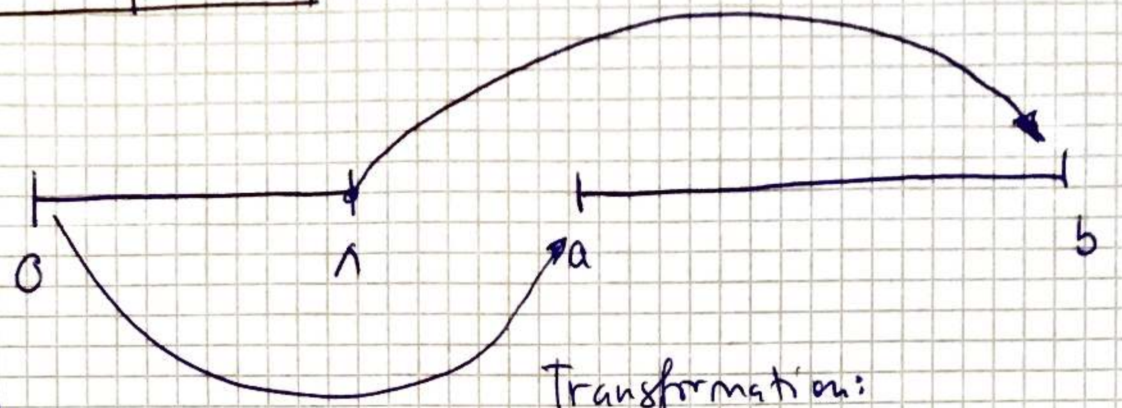
$\boxed{7.15. \ (p. 523)}$



$\boxed{1.}$

Transformation:

$$\phi(\tau) := (1-\tau)a + \tau b$$

$$\Longrightarrow \quad \phi(\tau = 0) = a \checkmark$$
$$\phi(\tau = 1) = b \checkmark$$

$$\int_a^b f(t)\,dt \approx (b-a)\sum_{j=1}^{n} \hat{\omega}_j \hat{f}(\hat{c}_j) = \sum_{j=1}^{n} \omega_j f(c_j)$$

with
$$c_j = (1-\hat{c}_j)a + \hat{c}_j b \; \left(= \phi(\hat{c}_j)\right)$$
$$= a + \hat{c}_j (b-a)$$
$$\omega_j = (b-a)\,\hat{\omega}_j$$

weights are scaled by the ratio of lengths of [a, b] and [0, 1]

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$\boxed{2}$ $\dfrac{\cosh z}{\sinh z} = \dfrac{(e^{z}+e^{-z})\frac{1}{2}}{\underbrace{(e^{z}-e^{-z})\frac{1}{2}}}$

We have to remove the singularity, i.e. $e^{z}-e^{-z}=0$!

$\Rightarrow e^{\pi i} = e^{-\pi i} = -1$

$e^{2\pi i} = e^{-2\pi i} = 1$

$\Rightarrow e^{z}-e^{-z}=0$ iff $z = \dots -2\pi i, -\pi i, 0, \pi i, 2\pi i, \dots$

$= \pi \cdot i \underbrace{\mathbb{Z}}_{\text{integral numbers}}$

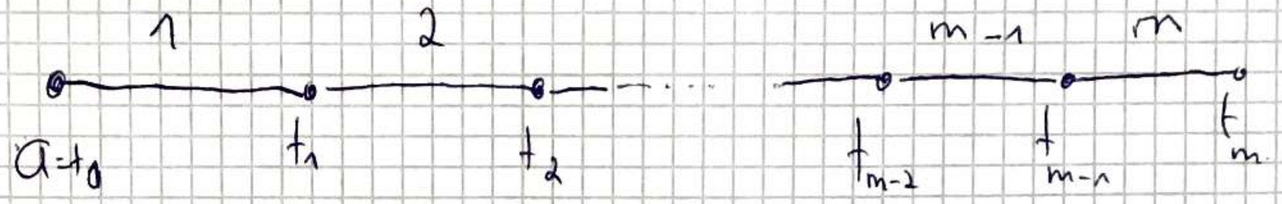
$\Rightarrow$ Domain of analycity:

$A := \mathbb{C} \setminus \{\pi i \mathbb{Z}\}$.

3

- m intervals
- on each interval $j = 1, \dots, m$: $f \in P_k$.

$\Rightarrow$ $m \cdot (k+1)$ degrees of freedom.



Let $F_{\ell,k} := \{ f \in C^{\ell}[a,b] : f_{|[x_{j-1}, x_j]} \in P_k, j = 1, \dots, m \}$.

Then: $$\dim F_{\ell,k} = \underbrace{m \cdot \dim P_k}_{\text{degrees of freedom}} - \underbrace{\# \{ C^{\ell} \text{ continuity constraints} \}}_{\text{linear constraints}}$$

(p. 418, Thm. 5.5.2).

$$\dim F_{0,k} = m(k+1) - (m-1) = \underline{\underline{mk+1}}$$

$$\begin{aligned} \dim F_{k,k} &= m(k+1) - (m-1)(k+1) = \\ &= mk + m - (mk + m - k - 1) \\ &= \underline{\underline{k+1}} \end{aligned}$$

$$\begin{aligned} \dim F_{k-1,k} &= m(k+1) - (m-1) k \\ &= mk + m - mk + k \\ &= \underline{\underline{m+k}} \end{aligned}$$

$\boxed{4b}$

Corollary 11.4.12: (p. 751)

RK-SSM is consistent with the ODE $\dot{y} = f(t, y)$ iff

$$\sum_{i=1}^{s} b_i = 1$$

$$\Rightarrow \gamma = \frac{1}{3}$$

$$\Rightarrow \sum_{i=1}^{3} \gamma = 1$$